Burning Rate Correlation with Effect of Thermal Radiation Considered

Northwestern Polytechnical University, Xian 710072, People's Republic of China
and
Feng Ren§
Shaanxi Institute of Power Machinery, Xian 710072, People's Republic of China

In this paper, a simplified model with the effect of thermal radiation considered is proposed for the composite solid propellant combustion. From the model, a semiempirical method is deduced to predict the correlation of propellant burning rates in a combustion bomb, a subscale motor, and a full-scale motor. The burning rates predicted with application of the method are in good agreement with those measured. The method possesses the advantage of being simple in calculation and of requiring less experiments.

Nomenclature

A = Arrhenius pre-exponential factor
a = correlation factor
B = mean effective radius of combustion chamber
C = specific heat
D_0 = motor diameter
d = grain web thickness
E = activation energy or emissivity
J = burning rate ratio (r/r_0)
M = molecular weight of gaseous species
$\dot{m}$ = mass flux
n = percent of Al in propellant, %
p = pressure
Qs = heat released by condensed phase reaction on burning surface
q = radiant flux
R = universal gas constant
r = burning rate
T = temperature
x = coordinate
β = relectivity of burning surface to incident radiation
λ = conductivity
ρ = density
σ = Stefen-Boltzmann constant

Subscripts
f = flame or combustion product
fo = adiabatic
g = gaseous phase
o = baseline or surrounding
p = Al_2O_3 or propellant
s = burning surface or condensed phase

I. Introduction

THE burning rate of solid propellants is an essential parameter for the design of solid rocket motors. The precision of burning rate determination affects directly the solid rocket motor performance prediction. Therefore, it is necessary to determine accurately burning rates of propellants. Up to now, there have been three kind methods to measure burning rates, i.e., a combustion bomb, a subscale motor, and a full-scale motor. However, the burning rate of the same propellant measured at the same pressure usually increases gradually from the combustion bomb to the full-scale motor. Ramamurthi[1] indicates that one important factor responsible for the difference in burning rates is the radiative heat transfer. The radiative heat transfer of combustion products of solid propellants has attracted quite a number of researchers in recent years.[2-6]

In the combustion bomb, the radiant heat loss from the flame and combustion products of propellant strand to the internal wall of the bomb leads the product temperature to decrease, which results in the decrease of strand burning rate. On the other hand, in a solid rocket motor with an internally burning grain, not only is there no radiant heat loss from combustion products but also there is radiant heat transfer from the high temperature products to the burning surface, which elevates the surface temperature to increase burning rates. Because the emissivity of gaseous species and Al_2O_3 particles in combustion chamber increases with increasing motor sizes, the radiant heat flux to the burning surface increases with the sizes, which results in the burning rate increase in the larger rocket motor. Therefore, studying the thermal radiation effect on burning rates is necessary for estimating precisely the burning rates in the full-scale motor.

Horton et al.[7] and Coates et al.,[8] supported by experimental results concerning the effect of external thermal radiation on burning rates of solid propellants at lower pressures, are of the opinion that the effect of thermal radiation comes from the increase of gaseous phase temperature during propellant combustion. As actual high pressure exists in an operating solid rocket motor, the burning rates predicted by Ramamurthi[1] with the use of Coates' model do not agree well with those experimentally determined.

In this paper, on the basis of the condensed phase combustion model of composite solid propellants, the effect of thermal radiation is considered to increase the burning surface temperature to elevate the burning rates. A semiempirical method for the correlation prediction of the three kinds of burning rates measured in a combustion bomb, a subscale motor, and a full-scale motor is obtained from the simplified combustion model.

II. Theoretical Model

The authors have studied experimentally the effect of external thermal radiation on the burning rates of composite

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*Lecturer, Department of Astronautical Engineering. Member AIAA.
†Senior Engineer, Department of Astronautical Engineering.
‡Professor, Department of Astronautical Engineering.
§Engineer.

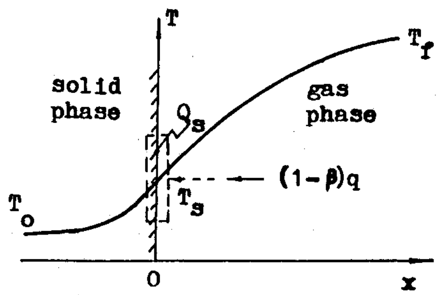

Fig. 1 Monodimensional combustion model.

solid propellants at higher pressures.[9] The results indicate that the external thermal radiation changes the burning rate through affecting the chemical reaction, especially the condensed phase reaction on the burning surface, during propellant combustion. Therefore, the effect of thermal radiation on the condensed phase burning process is significant. BDP (Beckstead, Derr and Price) model for the composite solid propellant combustion possesses the opinion that the condensed phase pyrolytic reaction is the dominant process during propellant burning. In the model, the burning surface temperature T_s is an important parameter. The authors deem that under the effect of thermal radiation on the burning surface, the energy balance on surface varies, so that T_s increases to lead the burning rates to increase. On the basis of the above mechanism and with reference to the schematic shown in Fig. 1, the following assumptions are made in this paper: 1) monodimensional; 2) one-step, irreversible gasification process on the burning surface described by Arrehnius' pyrolysis law; 3) constant physical parameters of propellant and gaseous phase, thermal perfect gas; 4) heat released in condensed phase reaction not affected by the thermal radiation; and 5) gray-body combustion products and the burning surface.

Under this set of assumptions, the burning process of propellant affected by thermal radiation is described by the following equations.

Mass flux on the burning surface

$$\dot{m} = A_s \exp\left(-\frac{E_s}{RT_s}\right) \tag{1}$$

Energy equation within condensed phase

$$\lambda_s \frac{d^2T}{dx^2} - \dot{m}C_s \frac{dT}{dx} = 0 \tag{2}$$

Energy balance equation on the burning surface

$$\left(\lambda_s \frac{dT}{dx}\right)_{0-} = \left(\lambda_g \frac{dT}{dx}\right)_{0+} + \dot{m}Q_s + (1-\beta)q \tag{3}$$

Boundary conditions

$$x = -\infty,\ T = T_0; \qquad x = 0,\ T = T_s \tag{4}$$

Integrating Eqs. (2–4) yields

$$T_s = T_0 + \frac{1}{C_s}\left[\left(\frac{\lambda_g}{\dot{m}}\frac{dT}{dx}\right)_{0+} + Q_s\right] + \frac{(1-\beta)q}{\dot{m}C_s} \tag{5}$$

When the thermal radiation exists, the characteristic time for gas phase will be shorter. Therefore, dT/dx increases and so does $\dot{m}$. Because the increases of dT/dt and $\dot{m}$ counteract one another in the term $(1/\dot{m})(dT/dx)|_{0+}$, it can be assumed reasonably that $(1/\dot{m})(dT/dx)|_{0+}$ is constant, i.e., that the heat feedback from gaseous phase to the burning surface $[(\lambda_g/\dot{m}C_s)(dT/dx)]_{0+}$ is only dependent on the combustion pressure approximately and is not affected by the thermal radiation. Therefore Eq. (5) is simplified to

$$T_s \approx T_{so} + \frac{(1-\beta)q}{\dot{m}C_s} \tag{6}$$

Substituting Eq. (6) into Eq. (1) gives

$$\dot{m} = A_s \exp\left(-\frac{E_s}{RT_{so}}\frac{1}{\{1 + [(1-\beta)q/\dot{m}C_sT_{so}]\}}\right) \tag{7}$$

As $[(1-\beta)q/\dot{m}C_sT_{so}] << 1$, so the following simplification for Eq. (7) is made

$$\frac{1}{1 + [(1-\beta)q/\dot{m}C_sT_{so}]} \simeq 1 - \frac{(1-\beta)q}{\dot{m}C_sT_{so}} \tag{8}$$

Therefore, the mass flux Eq. (7) is changed into

$$\dot{m} = A_s \exp\left(-\frac{E_s}{RT_{so}}\right)\exp\left(\frac{(1-\beta)qE_s}{\dot{m}C_sT_{so}^2R}\right) \tag{9}$$

When $q = 0$, $\dot{m}_0 = A_s \exp(-E_s/RT_{so})$; and $\dot{m} = \rho_p r$, Eq. (9) is reduced to

$$\dot{m} = \dot{m}_0 \exp\left(\frac{E_s}{RT_{so}^2C_s\rho_p}\frac{(1-\beta)q}{r}\right) \tag{10}$$

Letting the burning rate ratio $J = \dot{m}/\dot{m}_0 = r/r_0$, and correlation factor

$$a = \frac{E_s}{RT_{so}^2C_s\rho_p}$$

we obtain

$$J = \exp\left[a\frac{(1-\beta)q}{r_0J}\right] \tag{11}$$

From Eq. (11), it is found that J increases as the thermal radiant flux increases, and that the increment of burning rate is related to the physiochemical parameters of propellants (E_s, T_{so}, C_s, ρ_p, and so on) and baseline burning rate r_0 of solid propellant.

Now, a semiempirical method for the prediction of the burning rate correlation with effect of thermal radiation is obtained. For a composite solid propellant whose burning rates are measured in two small motors with different diameters, the radiant fluxes from combustion products to the burning surface in the two motors can be calculated. Therefore, the radiant flux difference ($q = q_2 - q_1$) can be determined. With solving Eq. (11), the correlation factor a for the propellant is obtained. The radiant flux in the third larger motor and the radiant flux difference between in the largest motor and in the smallest one can also be determined by calculating. With using this radiant flux difference, the correlation factor and the burning rate in the smallest motor, and with solving Eq. (11), the burning rate of propellant in the largest motor can be predicted.

In the application of the semiempirical method, the burning rate in a combustion bomb can be used to replace that in one small motor, in the case that there are not enough burning rate data in two smaller motors. But the semiempirical method is more suitable to the correlation prediction of burning rates in three motors with different diameters.

Two independent examples for the application of the semiempirical method to predicting the burning rate correlation have been made in the next section.

III. Theoretical Calculation and Experimental Results

A. Burning Rate Correlation Under the Effect of External Thermal Radiation

The experimental study for the effect of external radiation on the burning rates of composite solid propellants has been conducted in a combustion bomb with four view windows. A CO_2 laser with power of 37 W was used as the external radiation resource. The radiant fluxes are 7.2, 10.8, and 21.6 W/cm², respectively. The composite solid propellants used are listed in Table 1. The pressure of N_2 in the combustion bomb aranges from 1.0 to 4.0 MPa. The burning rates of propellant strands are determined by means of a high-speed camera. The results measured are shown in Figs. 2–4.

It is assumed that the reflectivity of burning surface to incident radiant beam from CO_2 laser equals zero. At a given pressure, with the use of baseline burning rate r_0 and the burning rate affected by external radiation r, and with solving Eq. (11), the burning rate correlation factor a is obtained for a given propellant.

For S01 propellant and at the baseline pressure of 3.0 MPa, $r_0 = 3.62$ mm/s, $r = 3.88$ mm/s, $J = r/r_0 = 1.072$, and $q = 21.6$ W/cm². Therefore $a = 0.012$. Substituting a and burning rates measured at 1.0, 2.0, and 4.0 MPa, respectively, into Eq. (11), J is calculated. As $r = Jr_0$, the burning rates of S01 at the pressures mentioned earlier under the effect of external radiation are predicted (see Fig. 2). The burning rates of S02, S03, and S04 affected by the constant radiant flux 21.6 W/cm² and the burning rates of S04 at 2.0 MPa affected by various radiant fluxes are also calculated in the same way (see Figs. 3 and 4). The baseline pressure for S02 is 3.0 MPa. As there are only three experiment points for S04 propellant, the middle one (2.0 MPa) is chosen as the baseline for S03 and S04 propellants. The results predicted agree with those measured.

B. Correlation of Burning Rates in a Combustion Bomb, a Subscale Motor, and a Full-Scale Motor

In this section, the propellant used is HTPB (11.65%), AP (69.5%), Al (18.5%), and additives (0.3–0.35%). In terms of the slight differences in the Al particle size and the concentration of additives, there are three kinds of formulas: S05A, S05B, and S05C. The subscale motor has a diameter of 152 mm, and the full-scale motor a diameter of 300 mm. The grain geometry in the two motors is star-shaped. Burning rates are determined in terms of the motor operating time and the web thickness of grain. The burning rates measured at 6.86 MPa in the combustion bomb and in the subscale motor are listed in Table 2. The parameters used in the example are listed in Table 3.

During the prediction of the correlation for burning rates in the bomb, the subscale motor, and the full-scale motor, it is the key step to determine precisely the thermal radiant fluxes from combustion products to burning surface.

The variation of emissivity of gaseous species with the motor radius described by the empirical formula, Eq. (12), is regressed from the data in Ref. 1 in which the chamber pressure is close to that in the example

$$E_g = 1 - \exp(-0.002071B) \quad (12)$$

The empirical formula for the emissivity of Al_2O_3 is quoted from Ref. 10

$$E_p = 1 - \exp\left(-12.943 \frac{n}{16} \rho_g B\right) \quad (13)$$

Table 1 Propellants used in the experiment

	S01	S02	S03	S04
Ingredients	AP-PU	AP-PU-AL	AP-HTPB	AP-HTPB-AL

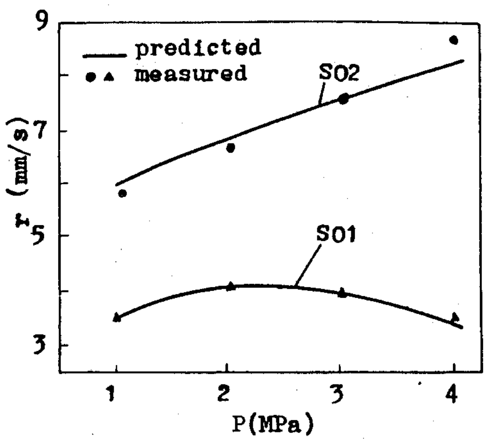

Fig. 2 Burning rates of S01 and S02 at external radiant flux 21.6 W/cm².

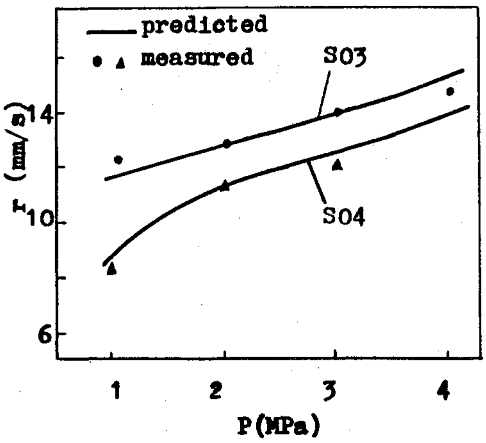

Fig. 3 Burning rates of S03 and S04 at external radiant flux 21.6 W/cm².

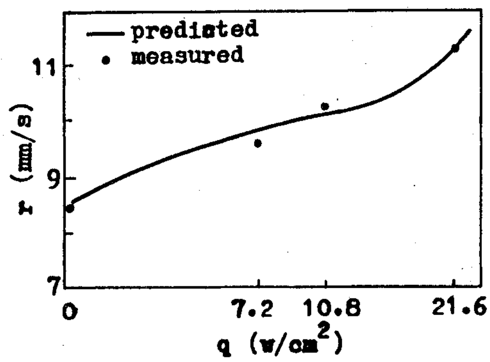

Fig. 4 Burning rates of S04 at various external radiant fluxes (2.0 MPa).

Where $B = 0.5(D_0 - d)$ (cm). The pressure dependence in Eq. (13) appears in the gas phase density. The Al_2O_3 particle emissivity [Eq. (13)] from Ref. 10 is suitable to the region from the combustion chamber to nozzle throat. In considering the radiative transfer within solid rocket motor in this example, the emphasis is laid on the radiant flux variation due to the different chamber diameters. As is well known, in the gas phase region near the burning surface (called as gas phase reaction region whose thickness is very thin), the products of burning aluminum droplets interchange radiation with the burning surface. This process is much more complicated. In this paper, this process is not considered in detail because for the motors with different diameters operating at the same pressure, the burning of aluminum droplets is subjected to the same mechanism, i.e., there is the same distribution of

Table 2 Results in combustion bomb and in subscale motor

Propellant	S05A	S05B	S05C
r(strand, mm/s)	9.44	10.06	9.74
r(Φ152 motor, mm/s)	10.41	10.75	10.66
q_1(W/cm^2)	141.00	141.66	141.33
q_2(W/cm^2)	369.64	369.66	369.65

Table 3 Parameters used in calculation

$B_{\Phi 152}$	6.975 cm	p	6.86 MPa
$B_{\Phi 300}$	12.15 cm	T_{fo}	3450 K
C_g	1.254J/g · K	T_s	700 K
E_f[a]	0.2	T_0	293 K
E_s	0.8	ρ_p	1.75 g/cm^3
M	28.14	β	0.0

[a]E_f in combustion bomb determined in terms of the strand size, the internal diameter of the bomb, and Eq. (13).

the composition of burning aluminum droplets near the burning surface. Therefore, the radiative transfer from the products of burning aluminum droplets is nearly the same for different diameter motors in the gas phase reaction region. So out of the gas phase reaction region, the variation of radiative transfer of Al_2O_3 particles with different motor diameters is the main radiation contribution that is considered emphatically. And out of the region, the emissivity of Al_2O_3 from Ref. 10 is valid. Furthermore, the application of this semiempirical method for the burning rate correlation prediction only requires the radiant flux differences among the motors with different diameters, which is a relative value, and does not require the absolute value of radiant flux in a definite diameter motor. Therefore, Eq. (13) is suitable to the situation in the example.

In terms of the thermal radiant heat transfer theory,[11] the emissivity of the mixture of gaseous species and particles is given approximately by the following equation

$$E_f = 1 - (1 - E_g)(1 - E_p) \tag{14}$$

Substituting Eq. (12) and Eq. (13) into Eq. (14) yields

$$E_f = 1 - \exp\left[-\left(12.942\frac{n}{16}\rho_g + 0.002071\right)B\right] \tag{15}$$

For the motor with internally burning grain, the radiant heat transfer between the combustion products to the burning surface is considered as in the situation of two infinite parallel planes, i.e.,

$$q = \sigma\frac{T_{fo}^4 - T_s^4}{\dfrac{1}{E_s} + \dfrac{1}{E_f} - 1} \tag{16}$$

In the combustion bomb, the radiant heat loss from the combustion products to the internal wall of bomb results in the decrease of temperature of combustion products. Therefore, two kinds of radiant heat transfer are considered.

1) The radiant heat transfer between combustion products and burning surface is taken as two infinite parallel planes, i.e.,

$$q_1 = \sigma\left[\frac{(T_f^4 - T_s^4)}{\left(\dfrac{1}{E_s} + \dfrac{1}{E_f} - 1\right)}\right] \tag{17}$$

2) The radiant heat transfer between combustion products and the internal wall of bomb is

$$q_{12} = \sigma E_f(T_f^4 - T_o^4) \tag{18}$$

Table 4 Results for the full-scale motor

Propellant	S05A	S05B	S05C
ΔT, K	50.36	48.85	49.24
q_3, W/cm^2	490.21	490.26	490.24
r, measured, mm/s	11.25	11.37	11.36
r, predicted, mm/s	10.94	11.12	11.16
Error, %	2.76	2.20	1.76

The flame temperature is modified by the following equation:

$$T_{fo} - T_f = \frac{q_{12}}{C_g \dot{m}_o} \tag{19}$$

The modified flame temperature and radiative flux from combustion products to burning surface in the combustion bomb (q_1) are obtained by solving Eqs. (17–19); q_1 for the three propellants are listed in Table 2.

In the subscale motor, using the measured burning rates and solving Eqs. (6) and (16), the modified burning surface temperature and the radiative flux (q_2) are determined; q_2 for the three propellants are also listed in Table 2. With application of q_1, q_2, and the burning rates measured in the combustion bomb and in the subscale motor listed in Table 2, the burning rate correlation factors for the three propellants are obtained by solving Eq. (11).

Because the operating conditions in the subscale motor are more similar to those in the full-scale motor than those in combustion bomb, and because there is some error in the calculation of radiative transfer in the combustion bomb (Ref. 12), the subscale motor is used as the baseline for the prediction of burning rates in the full-scale motor. By solving the simultaneous equations (Eqs. 6, 11, and 16) with a computer program, the increment of burning surface temperature from that in the subscale motor (ΔT), the radiative flux q_g, and predicted burning rates in the full-scale motor are gained. Table 4 lists ΔT, q_g and the burning rates measured and predicted. The radiative fluxes are in accord with Brewster's results from the two-flux radiant transfer model.[6] From Table 4, it is obvious that the burning rates predicted with the semiempirical method are in good agreement with those measured. The error is less than 3%.

IV. Conclusion

A simplified condensed phase combustion model for composite solid propellants with the effect of thermal radiation considered has been proposed with the assumption that the thermal radiation from combustion products to the burning surface comes from the increase of burning surface temperature. From the model, the authors have deduced a semiempirical method with which the correlation of burning rates in a combustion bomb and solid rocket motors with different diameters are predicted. If the burning rates in the two kinds of subscale motors with different sizes are measured, the burning rate in a rocket motor whose diameter is larger than those of the two motors could also have been predicted by the semiempirical method, and the prediction precision depends on determining radiant fluxes from combustion products to the burning surface in different rocket motors.

Examples for the application of the method have been made for four kinds of composite solid propellants under the effect of external radiation and for the burning rate correlation prediction of three kinds of HTPB aluminized propellants in a combustion bomb, a subscale motor, and a full-scale motor. The results predicted are in good agreement with experiments. The semiempirical method possesses the advantage of being simple in calculation and of requiring less experiments, which is useful in the design of solid rocket motors.

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